Hybrid Approach of Radial Basis Function and Domain Decomposition Method based on FEM with Different Shape of sub-domains for Electromagnetic Problems

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A novel approach, radial basis function (RBF) mixed with domain decomposition method (DDM) based on Galerkin finite element method (FEM) has been introduced in our previous work. The proposed method divides the computational domain into a series of rectangular sub-domains, and each sub-domain is taken as a separate calculation area to get the solution expression and shape function by using the point interpolation based on RBF. Then, sub-domains are taken as elements of the Galerkin FEM to approximate the entire solutions. The hybrid approach has been proved as a valid method, and inherited advantages of RBF and FEM. Nevertheless, the shape of sub-domain needs to be rectangular in that work and there are many different shapes in practical problems. More flexible form can be better adapted to electromagnetic problems. Triangular and cubical sub-domains will be investigated, respectively. In order to verify the improved method, several numerical examples including 3-D electromagnetic problem will be computed.

***Index Terms*—Numerical analysis, finite element analysis, numerical simulation, radial basis function (RBF).**

I. Introduction

Previously, we presented a hybrid method of RBF and FEM [1], which divides the computational domain into a series of sub-domains and uses RBF collocation method to construct the shape function in each sub-domain for Galerkin FEM approximating of the entire region. The total coefficient matrix becomes sparse in this process, even though the RBF is a kind of global basis function. The reason is that the sub-domain is set as the support domain of RBF, so the interpolation points based on RBF is no effective out of the sub-domain [2], [3].

Therefore, this new method is an improved DDM combined RBF collocation method equivalently, and inherits its advantages which the traditional RBF method doesn't have. For example, discontinuities in the derivatives of the fields near materials interfaces are not easily modeled for simple RBF as an infinitely smooth function [4], [5], while the proposed method get a high precision solution by dividing the different medium into different domains. Because of sparse coefficient matrix, novel approach has a greater range of useful shape parameter c and easier to find [6].Moreover, the global solution obtained by the proposed method is based on FEM, so the complex boundary conditions have the same easy treatment as FEM. Nevertheless, this process for ordinary DDM combined RBF collocation method is complex [5], let alone traditional RBF [7].

The new method can be considered as a special FEM. The shape function expressed by RBF is nonlinear approximation and will be more close to the actual result of element, compared with ordinary FEM based on linear shape function. Hence, the proposed method is less node and mesh setting, more flexible mesh, and high efficient fitting [1].

In the previous paper, only rectangular sub-domains are treated [1]. Different shape of sub-domains can also be used when the boundary of domains have nodes set. Now, we change the shape of domains for more different electromagnetic problems. Triangular and cubical sub-domains will be studied, and this change is proved to be effective and with high precision. In order to solve 3-D problem, we adopted cubic sub-domains and RBF of 3-D Cartesian coordinate system. Several numerical examples will be introduced to demonstrate these conclusions.

II. Hybrid approach

Use RBF collocation method to approximate the field with N_i collocation nodes set respectively. Then, the solution of each element $\boldsymbol{U}_i$ can be expressed by RBF as

$$\boldsymbol{U}_i = \sum_{j=1}^{N_i} a_j Q_j(x) = \boldsymbol{Q}_i(x)^T \boldsymbol{a}_i \quad (1)$$

where $\boldsymbol{a}_i$ is coefficient vector undetermined, and $\boldsymbol{Q}_i(x)$ is the RBF vector form. Define these equations below

$$\boldsymbol{u}_i = \left[U(x_1)_i \cdots U(x_{N_i})_i\right]^T \quad (2)$$

$$\boldsymbol{A} = \left[\boldsymbol{Q}_i(x_1) \cdots \boldsymbol{Q}_i(x_{N_i})\right]^T \quad (3)$$

The solution $\boldsymbol{U}_i$ can be rewritten as

$$\boldsymbol{U}_i = \boldsymbol{Q}_i(x)^T \boldsymbol{a}_i = \boldsymbol{Q}_i(x)^T \boldsymbol{A}^{-1}\boldsymbol{u}_i = \boldsymbol{\varphi}_i(x)\boldsymbol{u}_i \quad (4)$$

Then, the vector form of the shape function $\boldsymbol{\varphi}_i(x)$ is expressed as

$$\boldsymbol{\varphi}_i(x) = \boldsymbol{Q}_i(x)^T \boldsymbol{A}^{-1} \quad (5)$$

Obtain the shape expression by RBF, and then we could get linear equations based on Galerkin FEM. Transform the governing equations into the corresponding integral forms based on the principle of Galerkin's weighted residual. $\boldsymbol{u}_i$ as undetermined coefficient will be calculated from the linear equations.

III. NUMERICAL EXAMPLES

To clearly illustrate the efficiency of different sub-domains, we study two numerical examples.

A. *Triangular sub-domains*

The potential distribution of square metal box has been calculated by the hybrid method with high accuracy before. The governing equations are

$$\begin{cases} \Delta\phi = \dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2} = 0 \\ \phi\big|_{x=0,x=3,y=-1} = 0 \\ \phi\big|_{y=0} = \sin\dfrac{\pi x}{3} \end{cases} \tag{6}$$

Now, we still compute this mathematical model, but the difference is that the meshes are no longer rectangle. Fig. 1 demonstrates the node and grid distribution of the computational domain. The model of square metal box is partitioned into six triangular sub-regions $\Omega_1,\Omega_2,\Omega_3,\Omega_4,\Omega_5,\Omega_6$. Fig. 2 shows the results obtained from the proposed approach when the shape parameter c is 1.00. It can be seen that the solution by the proposed method is very close to the analytical solution and original hybrid method. Therefore, the novel approach also works with different shape of sub-domains.

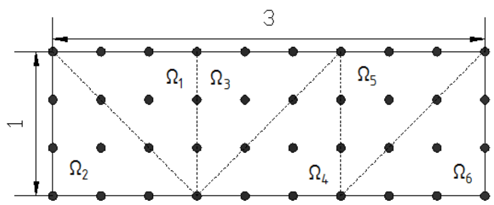

Fig. 1. The node and grid distribution

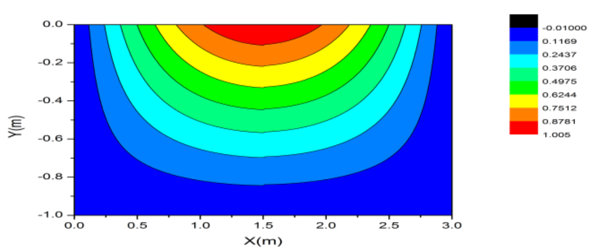

Fig. 2. The contour of potential by using the hybrid method

B. *Cubical sub-domains*

The novel approach can be used to solve 3-D electromagnetic problem. Not only cubic sub-domains, we have to introduce MQ-RBF in 3-D Cartesian coordinate system. The MQ-RBF is written as

$$Q_j(x) = Q_j(x,y,z) = \sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2 + c_j^2} \tag{7}$$

where c_j is a shape parameter which influences the fitting accuracy. (x_j, y_j, z_j) are the coordinates of interpolation point.

A cubic metal box as shown in Fig. 3 is computed. The electric potential of up wall is 1, and others are 0. Fig. 4 shows the result of potential distribution at y=0.5 from proposed method. As a benchmark problem, the analytical solution of the cubic metal box is well known. It can be seen that the results are in good agreement, so the new method is efficiency for 3-D electromagnetic problem. In order to fully verify the conclusion, more complex 3-D problems will be introduced in full paper.

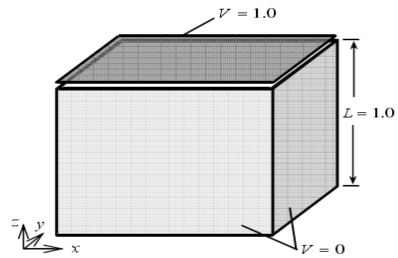

Fig.3. 3-dimensional cubical metal box

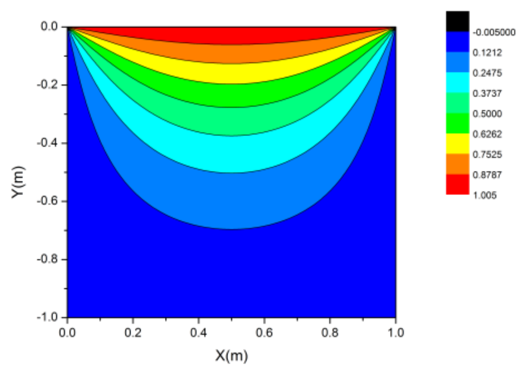

Fig.4. Equipotential linesat y=0.5 using hybrid method

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